

Mathematics Notes

Prepared by:

Muhammad Gshaqj Laskani

Teacher:

Sir Akhtar Malik

Lecture 1

BODMAS

B → Bracket

O → order or Exponents

like () (ii) { } (iii) []

D → Division

M → Multiplication

A → Addition

S → Subtraction

Q.No: 1 Simplify the

$$210 \div [24 - \{15 - (4 \times 3)\}]$$

Solution

Step 1: Bracket small solve

$$= 210 \div [24 - \{15 - 12\}]$$

Step 2: Middle Bracket

$$= 210 \div [24 - 3]$$

Step 3: Big Bracket

$$= 210 \div 21$$

Step 4: Division

$$= \frac{210}{21} = 10 \text{ Ans}$$

ii) Simplify the $11 - 11 \times 11 + 11 \div 11$

Solution:

Here is no Bracket involve so we Follow "OMAS" Rule

$$11 - 11 \times 11 + 11 \div 11$$

Step 1: Division

$$11 - 11 \times 11 + 1 \quad \because 11 \div 11 = 1$$

Step 2: Multiplication

$$11 - 121 + 1 \quad \because 11 \times 11 = 121$$

Step 3: Addition

$$\begin{aligned} -121 + 11 + 1 &= -121 + 12 \\ &= \boxed{-109} \text{ Ans} \end{aligned}$$

iii) Simplify the:

$$\begin{aligned} &3 - (2^2 - 2 [3 - 16 \div 2]) \\ &= 3 - (2^2 - 2 [3 - 8]) \\ &= 3 - (2^2 - 2 [-5]) \\ &= 3 - (2^2 + 10) \\ &= 3 - (8 + 10) = 3 - (18) \\ &= 3 - 18 = \boxed{-15} \text{ Ans} \end{aligned}$$

iv) Simplify

$$\begin{aligned} &2 - \{- \{ - (-5 + 3) \} \} \\ &= 2 - \{- \{ - (-2) \} \} \\ &= 2 - \{- \{ 2 \} \} \\ &= 2 - \{-2 \} \\ &= 2 + 2 \\ &= 4 \end{aligned}$$

v) Simplify

(03)

$$\begin{aligned} & \sqrt{289} (\sqrt{256} + \sqrt{676}) \\ &= \sqrt{17^2} (\sqrt{16^2} + \sqrt{26^2}) \\ &= 17(16 + 26) \\ &= 17(42) \\ &= \boxed{714 \text{ Ans}} \end{aligned}$$

vi) Simplify

$$\begin{aligned} & \frac{6^4 - 6^3}{5} \\ &= \frac{6^{3+1} - 6^3}{5} \quad \because 6^3 \text{ taken common} \\ &= \frac{6^3(6^1 - 1)}{5} \\ &= \frac{6^3(\cancel{6} - 1)}{5} = 6^3 = 6 \times 6 \times 6 \\ &= \boxed{216} \end{aligned}$$

vii) Simplify

$$\begin{aligned} & \frac{5^8 - 5^6}{5^5} \\ &= \frac{5^{5+3} - 5^{5+1}}{5^5} \quad \because 5^8 = 5^{5+3} \\ &= \frac{5^5(5^3 - 5)}{5^5} = 5^3 - 5 \\ &= 5 \times 5 \times 5 - 5 \\ &= 125 - 5 \\ &= \boxed{120} \end{aligned}$$

Decimal

(04)

i) Simplify

$$0.2 \times 3$$

Step 1: First of all, to ignore the decimal and multiply the values simply

$$2 \times 3 = 6$$

Step 2: Secondly, ~~330~~ count the number of digits after decimals and add them together.

$$1 + 0 = 1$$

Step 3:

In last step right only 1 digit after decimal

$$\boxed{0.6 \text{ Ans}}$$

ii) Simplify

$$3 \times 0.3 \times 0.03 \times 0.003 \times 30$$

Step 1: Multiply the values

$$3 \times 3 \times 3 \times 3 \times 30 = 2430$$

Step 2: Count the number of digit after decimal

$$0 + 1 + 2 + 3 + 0 = 6$$

Step 3:

writes 6 digit after decimal

$$\boxed{0.002430 \text{ Ans}}$$

iii) Simplify

$$\sqrt{0.09}$$

$$= \sqrt{\frac{9}{100}} = \frac{\sqrt{9}}{\sqrt{100}} = \frac{\sqrt{3^2}}{\sqrt{10^2}} = \frac{3}{10} = 0.3$$

iv) Simplify the

~~0.08 x~~ $0.08 + 0.09 \div 0.1$

$$= 0.08 + \frac{0.09}{0.1}$$

$$= 0.08 + \frac{\frac{9}{100}}{\frac{1}{10}} = 0.08 + \frac{9}{100} \times \frac{10}{1}$$

$$= 0.08 + \frac{9}{10} = 0.08 + 0.9$$

$$= 0.98 \text{ Ans}$$

v) Simplify the

$0.9 \div 0.3 \times 0.3$

$= \frac{0.9}{0.3} \times 0.3$ $= \frac{9}{10} \times 0.3$ $= \frac{9}{10} \times \frac{3}{10}$ $= \frac{9}{10} \times \frac{3}{10} \times 0.3 =$	<p style="text-align: center;">دوسرا طر لقمہ</p> $0.9 \div 0.3 \times 0.3$ $= \frac{0.9}{0.3} \times 0.3$ $= 0.9$
---	---

$$0.3 \times 3 = 0.9$$

vi) Simplify the

$0.02 \div 20$

$$= \frac{0.02}{20} = \frac{2}{1000}$$

$$= \frac{2}{1000} \times \frac{1}{20}$$

$$= \frac{2}{1000} \times \frac{1}{20} = 0.001$$

vii) Simplify

$$\frac{0.6 + 0.6 + 0.6 + 0.6 + 0.6}{5}$$

$$= 0.6 \left(\frac{1+1+1+1+1}{5} \right)$$

$$= \frac{0.6(5)}{5} = \boxed{0.6 \text{ Ans}}$$

viii) Simplify

$$2.25 - 8.19 - 2.85 - 4.8 + 1$$

$$= 2.25 + 1 - 8.19 - 2.85 - 4.8$$

$$= 3.25 - 15.84 = \boxed{-12.59}$$

Polynomial

Formula:

Let we have the polynomial $ax^b = 0$

Here "a" is called co-efficient, "x" is

called base (variable) b is called power

$$(a^x)^y = a^{x \times y}$$

$$a^x \times a^y = a^{x+y}$$

$$a^x \div a^y = a^{x-y}$$

$$x^a = \frac{1}{x^{-a}}$$

$\ln =$ natural log

$$\ln(a^x) = x \ln a$$

$$\ln(1) = 0$$

$$\ln(0) = \infty \text{ infinity}$$

Question: 1

If $4^{2a+3} = 4^{a-1}$ Then Find the value of a

Solution:

$$4^{2a+3} = 4^{a-1}$$

\therefore Since base is same on both side of equation. So we equate power as follows

$$2a + 3 = a - 1$$

$$2a - a = -1 - 3$$

$$a = -4$$

Question 2

If $64^{12} = 2^{a-3}$ what is the value of a

$$64^{12} = 2^{a-3}$$

$$(2^6)^{12} = 2^{a-3}$$

$$2^{72} = 2^{a-3}$$

\therefore base is same so Equate the Power

$$72 = a - 3$$

$$a = 72 + 3$$

$$a = 75$$

$$Q \neq 3$$

Find the value of x

$$x = (8)^{4/3}$$

$$x = (2^3)^{4/3}$$

$$x = 2^{8 \times 4/3}$$

$$x = 2^4$$

$$x = 16$$

Ques #4

Find the value of x

$$3^{-x} = \frac{1}{27^{(3+x)}}$$

$$3^{-x} = 27^{-(3+x)}$$

$$3^{-x} = (3^3)^{-(3+x)}$$

$$3^{-x} = (3)^{-3(3+x)}$$

$$3^{-x} = 3^{-9-3x}$$

\therefore Since base is same so

we can equate the power.

$$-x = -9 - 3x$$

$$-x + 3x = -9$$

$$2x = -9$$

$$x = -9/2$$

$$x = -4.5$$

Question #5

Find the value of x

$$4^{x+3} = 8^{x-1}$$

$$(2^2)^{x+3} = (2^3)^{x-1}$$

$$2^{2(x+3)} = 2^{3(x-1)}$$

$$2^{2x+6} = 2^{3x-3}$$

\therefore base is same so Equate the power

$$2x+6 = 3x-3$$

$$6+3 = 3x-2x$$

$$9 = x$$

vii) Simplify the

$$p^{-2} \times p^1 \times p^8$$

As we know that

$$a^x \times a^y \times a^z = a^{x+y+z}$$

So:

$$\begin{aligned} & p^{-2} \times p^1 \times p^8 \\ &= p^{-2+1+8} \\ &= p^{-2+9} = \boxed{p^7} \end{aligned}$$

(Question (viii))

If $8 \times 8 = 4^x$ what is value of x

Solution:

$$8 \times 8 = 4^x$$

$$64 = 4^x$$

$$4^3 = 4^x$$

\therefore Base is same so Equate the power

$$\boxed{x = 3}$$

Number System

$p = 460$ یہ سارے اعداد کیس سے کرتے ہیں

Highest common Factor (H.C.F)

Find all Prime factor of both numbers.

writes both number as a multiplication of Prime numbers.

Find Factor are repeating in both numbers and multiplying them to H.C.F.

Question #1

Find H.C.F of 24 and 36

$$24 = 2 \times 2 \times 2 \times 3$$

$$36 = 2 \times 2 \times 3 \times 3$$

$$= 2 \times 2 \times 3 = 12$$

$$\begin{array}{r|l}
 & 24 \quad 36 \\
 \hline
 2 & 12 \quad 18 \\
 \hline
 2 & 6 \quad 9 \\
 \hline
 3 & 3 \quad 3 \\
 \hline
 & 1 \quad 1 \\
 \hline
 & 1 \quad 1
 \end{array}$$

Question #2

Find Common Factor of 90, 70

$$90 = 2 \times 3 \times 3 \times 5$$

$$70 = 2 \times 5 \times 7$$

$$= 2 \times 5 = 10 \text{ H.C.F}$$

$$\begin{array}{r|l}
 2 & 90 \\
 \hline
 3 & 45 \\
 \hline
 3 & 15 \\
 \hline
 5 & 3 \\
 \hline
 & 1
 \end{array}
 \quad
 \begin{array}{r|l}
 2 & 70 \\
 \hline
 5 & 35 \\
 \hline
 7 & 5 \\
 \hline
 & 1
 \end{array}$$

Formulas

(11)

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

Question

If $x=2$ Find $[(x^2 + \frac{1}{x^2}) - (x + \frac{1}{x})]$

Solution:

$$= [(x^2 + \frac{1}{x^2}) - (x + \frac{1}{x})]$$

= putting the value of $x=2$

$$= [(2^2 + \frac{1}{2^2}) - (2 + \frac{1}{2})]$$

$$= [4 + \frac{1}{4} - 2 + \frac{1}{2}]$$

$$= [2 + \frac{1}{4} - \frac{1}{2}]$$

$$= \frac{8+1-2}{4}$$

$$= \frac{7-2}{4}$$

$$= \frac{7}{4}$$

Q: NO # 2

Find the value of $(a^2 + \frac{1}{a^2})$

$$(a + \frac{1}{a})^2 = 100$$

$$(a)^2 + \left(\frac{1}{a}\right)^2 + 2(a)\left(\frac{1}{a}\right) = 100$$

$$a^2 + \frac{1}{a^2} + 2 = 100$$

$$a^2 + \frac{1}{a^2} = 100 - 2$$

$$a^2 + \frac{1}{a^2} = 98$$

Question #3

The Sum of two number is 40 and their difference is 4. Find numbers

$$x + y = 40 \quad \text{---} \quad \textcircled{1} \text{ Equation}$$

$$x - y = 4 \quad \text{---} \quad \textcircled{2} \text{ Equation}$$

Added both the Equation we get

$$2x = 44$$

$$x = \frac{44}{2}$$

$$x = 22$$

Median

Arrange the Data in Ascending or Desending Form and Then Count the number of Data Point

If there are **odd number** of data point Then middle value is median

Question #4

Find the median of the following Data:

Q: The Average of two number is A.

If one number is x, what will be other number

Solution:

x of 2 number = A

$$\frac{x_1 + x_2}{2} = A$$

$$x + x_2 = 2A$$

$$x_2 = 2A - x$$

Q: If $20(x+y) = 70$. Then what is the Average of x and y?

Solution

$$20(x+y) = 70$$

$$x+y = \frac{70}{20} = \frac{7}{2}$$

Divide both side with 2

$$\frac{x+y}{2} = \frac{7/2}{2}$$

$$\frac{x+y}{2} = \frac{7}{4}$$

Mode

The most repeated value in a given set of Data points is called Mode

Q: Find the Mode of Following DATA:

7.5.8.4.8.5.8.5.5.7.5

Solution

Mode = 5

Range

The difference of maximum value and minimum value in a given set of data points is called Range.

Range = Maximum value - Minimum value

Q: Find the range of following Data:

13.9.2.4.1.8.7.3.11

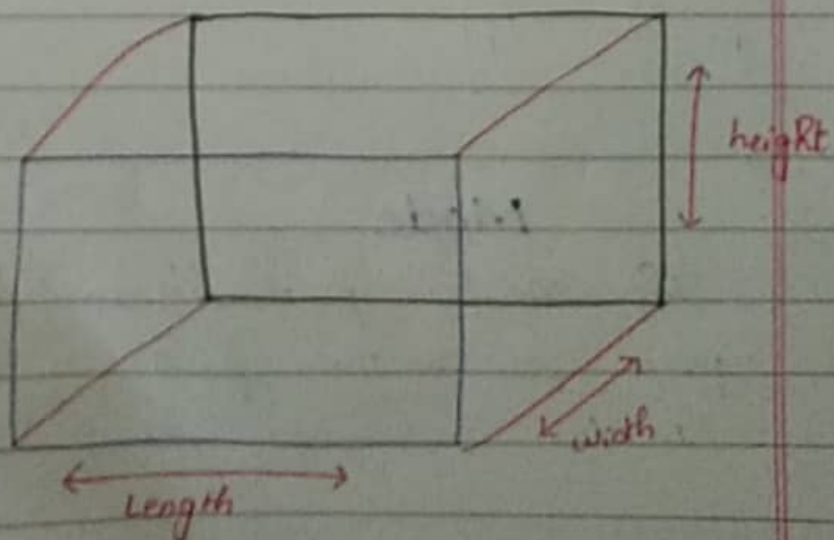
Range = $13 - 1$
 $= 12$

Geometrical Figures Problem

Formula:

Area of rectangle = $A = \text{Length} \times \text{width}$

Perimeter of rectangle = $p = 2(\text{Length} + \text{width})$



24, 45, 37, 32, 29, 35, 33, 45, 47

Solution

First Arrange the Data in Ascending Form:

24, 29, 32, (33), (35), 37, 45, 45, 47

So we have odd number and middle value will be median.

$$\text{median} = 45$$

Q #2

Find the median of Following Data

5, 5, 7, 8, 1, 2, 5, 6, 4, 9

Solution

place them in Ascending order

1, 2, 4, 5, 5, 5, 6, 7, 8, 9

we have even number of data points
So Average of middle two value will be median

$$\text{Median} = \frac{5+5}{2} = \frac{10}{2} = 5$$

Average

$$\text{Average} = \frac{\text{Sum of all values}}{\text{number of values}}$$

Question

Find the mean of the following Data

95, 87, 84, 84, 60

Solution:

$$\begin{aligned} \text{mean/Average} &= \frac{95+87+84+84+60}{5} \\ &= \frac{410}{5} \end{aligned}$$

$$\text{Average} = 82$$

Q: Find the Average of prime number b/w 30 and 50 are as follows

Solution

Prime number 30 and 50 = 31, 37, 41, 43, 47

$$\begin{aligned} \text{mean/Average} &= \frac{31+37+41+43+47}{5} \\ &= \frac{199}{5} = 39.8 \end{aligned}$$

Q: Find the Average of Four multiple of Fives:

Solution:

We know that Four multiple of Five are AS = 5, 10, 15, 20

$$\begin{aligned} \text{Average} &= \frac{5+10+15+20}{4} \\ &= \frac{50}{4} = 12.5 \end{aligned}$$

Area of square = $A = (\text{Length of one side})^2$

Volume of cube = $V = (\text{Length of one side})^3$

Area of cuboid =

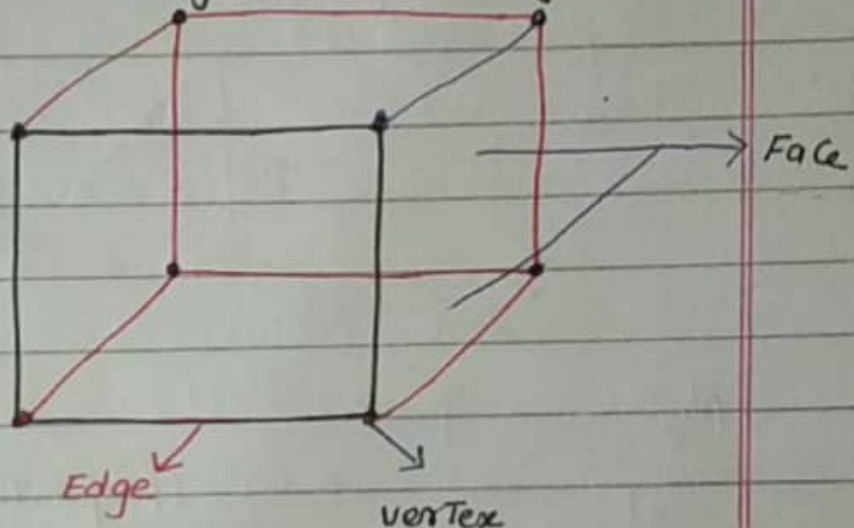
$$V = \text{Length} \times \text{width} \times \text{height}$$

on cube

vertices = 8

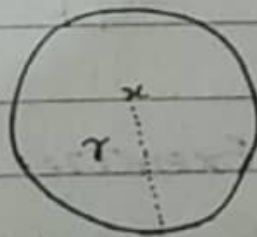
Edge = 12

Faces = 6



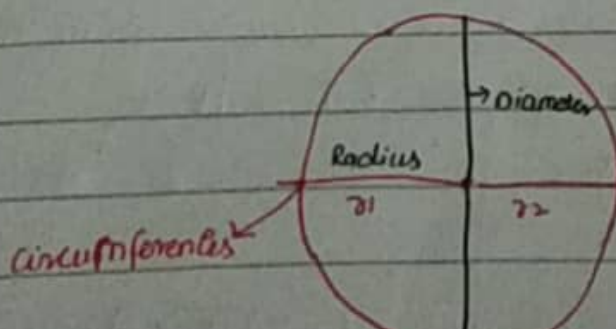
Surface Area of cube = $S = 6L^2$

Area of circle = $A = \pi r^2$



Circumference of circle = $C = 2\pi r$

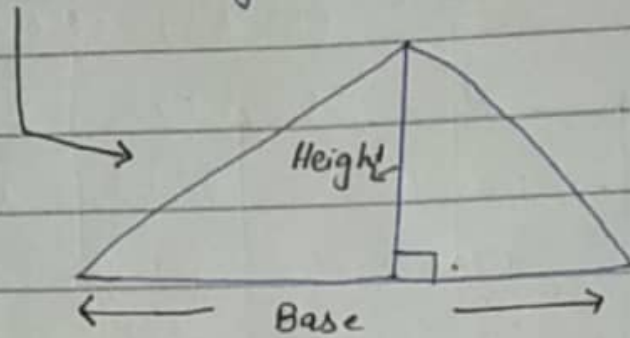
Diameter of circle = $d = 2r$



$$\text{Volume of Sphere} = \frac{4}{3} \pi r^3$$

$$\text{Surface Area of Sphere} = S = 4\pi r^2$$

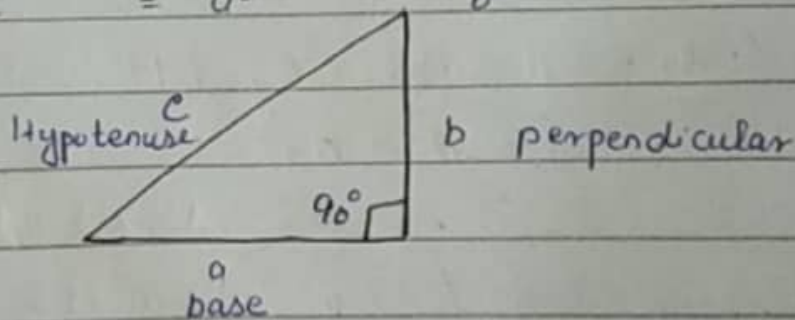
$$\text{Area of Triangle} = \frac{1}{2} \times \text{Base} \times \text{Height}$$



Pythagorean problem

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$$

$$c^2 = a^2 + b^2$$



Exercise

Q: The length of a garden is 7 km and its area is 35 km. Find its width?

Solution:

$$\text{Area} = \text{Length} \times \text{width}$$

$$35 = 7 \times \text{width}$$

$$\text{width} = \frac{35}{7} = 5$$

$$\text{width} = 5 \text{ km}$$

Q: Sum of all edge of cube is 24cm

Find the volume of cube

A cube has all side equal in length and passes 12 edge.

Length of one side = $\frac{24}{12} = 2 \text{ cm}$

Volume of cube = $V = (2)^3$

$V = 8 \text{ cm}^3$

Q: The Area of circle is 154 units

Find the Radius of circle

$A = \pi r^2$

$154 = \frac{22}{7} \times r^2$

$r^2 = \frac{7}{22} \times 154$

$r^2 = 49$

$(\sqrt{r})^2 = (\sqrt{49})^2$

$r = 7 \text{ units}$

Q: If the Radius of the circle is increased by two time. Find the change in Area of circle:

Solution:

$A = \pi r^2$

$r = 2r$

So

$A = \pi (2r)^2$

$A = \pi 4r^2$

$A = 4 (\pi r^2)$

$= 4 A$

Area Increased 4 times

Q: If perimeter of Rectangle is 68 Yards and width is 48 Feet. Find Length

Solution

We know that 1-Yard equal 3 Feet

$$P = 2(L + W), \text{ plus}$$

$$68 \times 3 = 2(L + 48)$$

$$204 = 2(L + 48)$$

$$\frac{204}{2} = L + 48$$

$$L + 48 = 102$$

$$L = 102 - 48 = \boxed{54 \text{ ft}}$$

Q:

Length of Rectangle is doubled and width is halved. Find change in Area

Solution:

$$A = L \times W$$

$$L = 2L$$

$$W = \frac{W}{2}$$

$$\text{SO } A = 2L \times \frac{W}{2}$$

$$A = L \times W$$

No change: